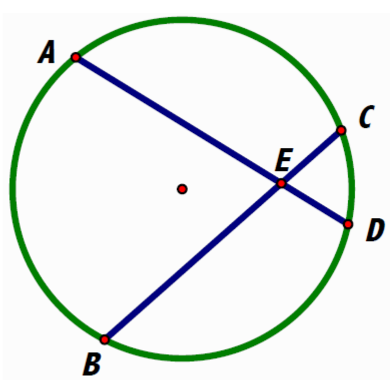
1 [10 points] A square has area $\frac{9}{4}\,\text{ft}^2$. What is the area of a circle inscribed in the square?

2 [10 points] In a right triangle the altitude to the hypotenuse divides the hypotenuse into two segments of length 4 and 9 units. Find the length of this altitude.

3 [10 points] Two chords AB and CD of a circle intersect at E. If $AE = 8$, $BE = 4$, and $DE = 2$, find CE.



4 [10 points] One triangle has sides of length 7, 5, and 4. The second triangle has sides of length 21, 15, and 12.

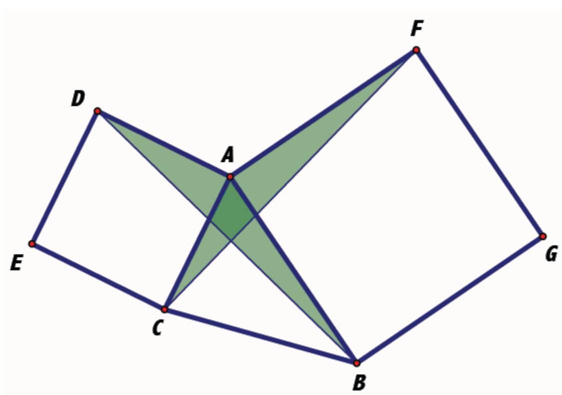
4.a Why are the triangles similar?

4.b Find the ratio of the area of the larger triangle to the area of the smaller triangle.

5 [20 points] Find the simplest possible expression for the sum of the exterior angles of a convex n-gon. (An exterior angle is a supplement of an interior angle.)

6 [20 points] Prove that the median to the hypotenuse in a right triangle is half as long as the hypotenuse. (A median is a segment connecting a vertex to the midpoint of the opposite side.)

7 [20 points] Squares have been constructed on the sides of ΔABC as shown below.



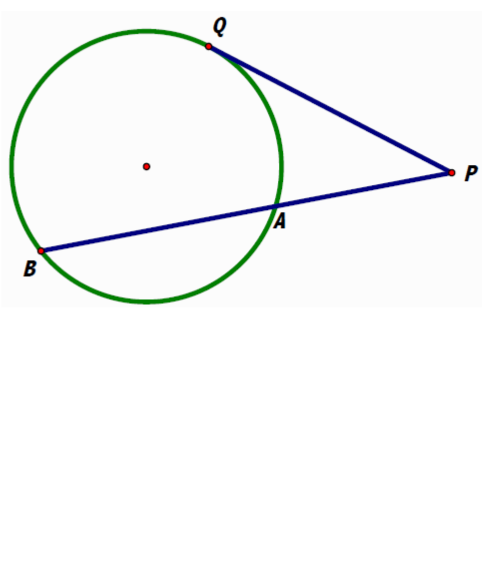
7.a Prove that the shaded triangles ADB and AFC are congruent and write the congruence between them symbolically.

7.b Prove that the segments BD and FC are perpendicular.

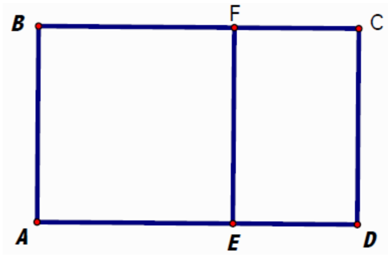
8 [20 points] State and prove the Law of Sines.

9 [20 points] Prove that the three angle bisectors of a triangle are concurrent (intersect in a single point).

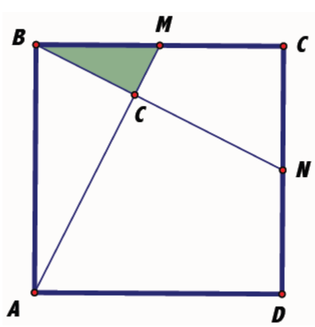
10 [20 points] A tangent and secant are drawn to a circle from an external point P. The tangent touches the circle at Q and the secant intersects the circle at A and B, as shown below. If $PA = 12$ and $AB = 15$, find PQ.



11 [20 points] A golden rectangle ($ABCD$ in the figure below) is a rectangle with the property that if it is dissected into two pieces–a square $ABFE$ and a rectangle $EDCF$–then the new rectangle $EDCF$ is similar to the original rectangle. Find the ratio of the longer side to the shorter side of the golden rectangle.



12 [20 points] In the figure below, $ABCD$ is a square and M and N are midpoints of two sides.



12.a Prove that the shaded triangle is similar to ΔABM and write the similarity statement symbolically.

12.b Find the ratio of the area of the shaded triangle to the area of the square $ABCD$. Justify your answer.

13 [20 points] Imagine that a rope is tightly stretched around the Earth's equator. Assuming that the Earth is a perfect sphere, imagine that the rope is taken off and 100 feet are added to it. The extended rope is magically put back around the equator so that the gap (marked x) created between the new circle and the equator is the same all around. Is the gap more or less than 10 feet? Justify your answer.